

these developments, creating a fruitful interaction between scientists. A good textbook on wavelets faces the difficult task of giving to the reader a clear idea of these pluridisciplinary aspects, while keeping a coherent exposition of the main theoretical results. In that respect, the *Ten Lectures* by I. Daubechies are a brilliant success.

This book describes the different aspects of wavelets, starting from the continuous wavelet transform and its relation with group theory, then concentrating on the redundant sampling of this transform that leads to frames, and finally describing the construction of orthonormal wavelet bases, their applications in functional analysis and operator theory, their related algorithms, and their possible generalization. This natural progression also corresponds to the scientific evolution of the author, who has made important contributions at every step. It gives a strong mathematical unity to this book. Moreover, the author regularly opens "windows" to the different related topics and applications: the auditory model, time-frequency localization, multiresolution approximation, subband coding schemes, subdivision algorithms, . . .

The mathematical prerequisite to the reading of this book is a basic knowledge of Fourier analysis and integration theory (some elementary results are recalled in the introduction). As a conclusion, I recommend this book to any scientist who wants to have a clear, yet not simplified, vision of wavelet theory.

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Y. MEYER, *Wavelets. Algorithms & Applications*, translated and revised by R. D. Ryan, SIAM, 1993, xi + 133 pp.

In a short time a large number of papers and books on wavelets have appeared. Each paper and book treats a very particular aspect of the theory or a special application. The novice is therefore faced with the task of going through a rather large amount of technical work. Seldom does he/she find historical pointers, how the theory grew, the variety of applications, and the potential in other areas of scientific interest.

In 1991, Yves Meyer gave a series of lectures on wavelets at the Spanish Institute in Madrid, Spain. The book under review is the result of these lectures (in fact, the English translation of Meyer's lectures). Without doubt the book shows that Meyer has fulfilled the objective of the Spanish Institute: *to present to a scientific audience coming from different disciplines, the prospects that wavelets offer for signal and image processing*, but even more, the book is highly recommended as a very readable introduction to the development from Fourier analysis to the present concept of wavelets.

The first chapter gives a short survey of signal processing and an intuitive view on the development of wavelets. Fourier analysis is the classical tool for stationary signals, but in order to analyze non-stationary or transient signals one needs different techniques including wavelets of *time-frequency* type and wavelets of *time-scale* type. Both types are then briefly explained. In Chapter 2, the author describes seven different origins of wavelet analysis. A problem with (pointwise) convergence of Fourier series led to the search for other orthonormal systems than the trigonometric system, in particular the Haar system and other systems of functions which allow the characterization of certain classes of functions, such as the Hölder spaces  $C^r$ . The study of multifractal structures stimulated wavelet techniques since wavelet coefficients can measure the average fluctuations of a signal at different scales, and this holds in particular for Brownian motion. Littlewood and Paley defined dyadic blocks of the Fourier series of a function  $f$  to characterize the norm  $\|f\|_p$ . Zygmund's search in extending the Littlewood-Paley result to  $n$ -dimensional Euclidean space led to the introduction of the *mother wavelet*  $\psi(x)$ . Other developments in the 1930's, such as the Franklin system and the wavelets of Lusin, are also important directions in the new theory of wavelets.

The Hardy spaces  $H^p$  also make their entrance and new names such as Weiss, Coifman, and Strömberg can be added to the list of relevant wavelet people.

Chapters 3 and 4 present time-scale algorithms for signal processing and image processing. Chapters 5–7 deal with time–frequency algorithms. In Chapter 3, quadrature mirror filters are introduced. They are used to decompose a signal  $f$ , sampled on a fine grid, into a trend and a sequence of fluctuations. The inverse transformation is calculated by the reconstruction property of quadrature mirror filters. The asymptotic behavior of the algorithm is given if the sampling grid becomes infinitely fine. Chapter 4 describes the pyramid algorithms of Burt and Adelson and other orthogonal and biorthogonal pyramid algorithms that calculate iteratively approximations of a given signal at different scales. The Gabor wavelets, treated in Chapter 5, introduced the search for a representation of a signal in time–frequency atoms. To this end, the Wigner–Ville transform is described, but this transform unfortunately did not lead to an algorithm for atomic decomposition. Gabor wavelets give an optimal localization in the time–frequency plane. The Malvar wavelets (Chapter 6) and the wavelet packets (Chapter 7) do not have this optimality, but they give us a whole universe of interesting orthonormal bases. The author then suggests an entropy criterion to select the algorithm and the wavelets that give an optimal decomposition.

In Chapter 8, the coding of an image using the zero-crossings of the wavelet transform is explained. Chapter 9 gives an application in the determination of the fractal exponent of the Weierstrass function (Riemann's example of a continuous function which is nowhere differentiable). Chapters 10 and 11 are not worked out in as much detail as the previous chapters and pose some challenges and open problems concerning the multifractal approach of turbulence and the hierarchical organization of galaxies and the structure of the universe.

The book is very pleasant to read. In contrast to the existing literature on wavelets, there are very few theorems and proofs in this text, and the emphasis is on a clear exposition explaining the underlying ideas and situating the various techniques and ideas in an historical context. This book is recommended reading.

LIEVE DELBEKE AND WALTER VAN ASSCHE

G. H. KIROV, *Approximation with Quasi-Splines*, Institute of Physics, 1992, vii + 247 pp.

This book is a collection of results in approximation theory which, as the author points out in the preface, closely follows his research interests. It serves two purposes: first, to introduce the reader to the properties and applications of quasi-splines, and second, to survey relevant optimal recovery results which provide the motivation for the development of quasi-splines.

Interest in quasi-splines appears to be confined to the author and his colleagues. The quasi-spline of order  $r$  generated by the information  $f^{(k)}(x_i)$ ,  $i = 1, 2, \dots, n$ ;  $k = 0, 1, \dots, r$  for the function  $f$  is

$$\Phi(x) = \sum_{i=1}^n \sum_{k=0}^r k_i(x) \frac{f^{(k)}(x_i)}{k!} (x - x_i)^k, \quad x \in [0, 1],$$

where the  $k_i(x)$  are a set of fundamental or basic quasi-splines that satisfy

$$\sum_{i=1}^n k_i(x) = 1, \quad x \in [0, 1].$$

Spline functions are the solution of many extremal problems, based on information  $f(x_i)$  at nodes or mesh points  $x_i$ ,  $i = 1, 2, \dots, n$ . Quasi-splines are developed as a generalization of spline functions. These generalized functions may be discontinuous at the nodes. Classical splines are a special (continuous) case when  $r = 0$ .